# EEP/IAS 118 - Introductory Applied Econometrics, Lecture 7

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## This Lecture

## Topics

- Review
- P-values
- Linear combination tests
- Testing multiple parameters

Assignments

- Problem Set 3 Due Wednesday
- Quiz 3 Tomorrow
- New Daily Assignments Posted

Midterm

• This is the last lecture that will be included on the Midterm (one week from today) - bring a calculator!

# Quiz 2

Let's review Quiz 2:

- Question 1: We have downward bias. Why?
  - $\beta_{age} > 0$
  - $\rho_{age,educ} < 0$
  - $\Rightarrow$   $Bias = \beta_{age} * \rho_{age,educ}$  is negative
- **Question 2:** Having a random sample will not fix any problem stemming from omitted variable bias.
- Question 3:

$$[10 - 2.262 * \frac{2}{\sqrt{10}}, 10 + 2.262 * \frac{2}{\sqrt{10}}]$$

## Problem Set 2

Let's review issues from PS2:

- Remember to specify "holding all else constant", when interpreting  $\hat{\beta}$  in a regression with multiple variables
- Be clear about your reasoning when inferring correlations between variables in OVB questions
- Exercise 2, Question 5: What should we infer from the fact that the coefficient on *pencil* was a similar magnitude as the coefficient on *computer*?
  - Since it is silly to think people are paid a higher wage for being able to use a pencil, this implies that there is a common OVB problem in these regressions. We therefore should question the coefficient on *computer*, as the omitted variables that biased our estimate for the pencil regression are likely to also bias our estimate for computers

# Specify Null and Alternative

We observe the average consumption on temptation goods (cigarettes, alcohol, and tobacco) for clients and non-clients of a microfinance institution (MFI).

 Want to test that the average consumption of temptation good is the same between non-MFI clients and MFI clients.
Call D the difference in mean consumption of the two groups.

Specify the null and alternative hypothesis

• Want to test that the average consumption of temptation good is higher among non-MFI clients. Call *D* the difference in mean consumption of the two groups.

Specify the null and alternative hypothesis

## Specify Null and Alternative

 Want to test that the average consumption of temptation good is the same between non-MFI clients and MFI clients.
Call D the difference in mean consumption of the two groups.

Specify the null and alternative hypothesis:  $H_0: D = 0, \quad H_1: D \neq 0$ 

• Want to test that the average consumption of temptation good is higher among non-MFI clients. Call *D* the difference in mean consumption of the two groups.

Specify the null and alternative hypothesis:  $H_0: D = 0, H_1: D > 0$  According to the 2010 U.S. Census (i.e., US population), the average household income was \$100,000. Suppose in the year 2015 you draw a random sample of two thousand U.S. households that has a sample mean of \$105,000 with a sample standard deviation of \$6,000.

We want to conduct a hypothesis test to evaluate whether average incomes in 2015 are larger than those in 2010. Write down the null and alternative hypotheses

# Specify Null and Alternative

We want to conduct a hypothesis test to evaluate whether average incomes in 2015 are larger than those in 2010. Write down the null and alternative hypotheses

 $H_0: \mu_{2015} = 100,000$  $H_1: \mu_{2015} > 100,000$ 

What were our clues?

- In description of 2010 data "i.e. US population"  $\rightarrow$  100,000 is the true population parameter, not an estimator
- There was no standard deviation given for the 2010 data, if there is a difference in means test you will need to calculate  $s^2$  for *both* samples

## Aside: Relationship Between CI and Hypo. Tests

We have talked about these procedures separately, but they are related

• The basic formula for a CI is:

$$[\hat{\theta} - c \cdot se(\hat{\theta}), \ \hat{\theta} + c \cdot se(\hat{\theta})]$$

• The basic formula for a test stat which we compare to c is:

$$\frac{\hat{\theta} - \theta_{H_0}}{se(\hat{\theta})}$$

If θ<sub>H<sub>O</sub></sub> falls outside of the the CI for a given size (e.g. 90%, 95%, 99%), then we would also reject at the same significance level that the null hypothesis is true

Selecting the significance level  $(\alpha)$  can be arbitrary

- Two reasonable people could choose different  $\alpha$  and reach different conclusions
- Is a t-stat of 1.94 really that different from a t-stat of 1.96?
- Binary reject / fail-to-reject obscures the continuous nature of the random draw

Instead we can examine the p-value of a test for an objective idea of the robustness of the result. What is a p-value?

Here are 2 definitions of p-values that say the same thing using different words:

- The p-value is the smallest significance level at which the null hypothesis would be rejected.
- The p-value is the probability of obtaining a value of the test statistic as extreme or more extreme than the one actually obtained from the sample under the null (i.e if the null is true).

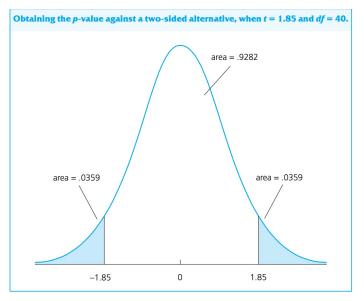
### **P-Values**

- P-values close to zero constitute strong evidence against the null  ${\cal H}_0$
- Large p-values close to one constitute weak evidence against the null *H*<sub>0</sub>.

**Example:** Suppose we calculate a test statistic of t=1.85, with 40 degrees of freedom (two sided). We can find the p-value

 $pvalue = P(|T| > 1.85|H_0) = 2P(T > 1.85) = 2(0.0359) = 0.0718$ 

#### P-value of 0.0718:



#### p-values

In this example, if the null were true (e.g.  $\beta=0$  or  $\mu=10,$  etc.), then:

- We would observe a t-stat as large as 1.85 about 7.2% of the time
- We would *fail* to reject the null at 5% significance level
- We would reject the null at 10% significance level

p-values are reported as the fourth column in the Stata output

## p-values in Stata

Source	SS	df	MS	Number of obs	=	526
+				F(3, 522)	=	94.75
Model	52.2939096	3	17.4313032	Prob > F	=	0.0000
Residual	96.0358418	522	.183976708	R-squared	=	
+				Adj R-squared	=	0.3488
Total	148.329751	525	.28253286	Root MSE	=	.42893
lwage	Coef.	Std. Err.	t P>	> t  [95% Co	onf.	Interval]
educ	.0912897	.0071232	0.	.000		
exper	.0094139	.0014493	6.50 0.	.000 .006566	37	.012261
	.0094139	.0014433	0.50 0.	.000 .000300		
female		.0376668		.000417593		2695996

- The p-value stata shows you is the p-value for the null hypothesis that  $\hat{\beta}=0.$  This is not necessarily the one you want
- We now know almost everything the regression output produces

## Sign, Size, and Significance

We can now address the last step in our interpretation of  $\hat{\beta}$  from regression output:

- In addition to sign and size, we need to comment at what level the coefficient is statistically significant
- Statistically significant means: we reject the null hypothesis that  $\beta$  is equal to zero
- We can determine this by looking at the p-value (or t-stat) for the coefficient in the regression output
- Language: " $\hat{\beta}$  is statistically significant at the 10% level, but not at the 5% level (p-value of 0.07)"
  - Stick to the standard significance levels (10%, 5%, 1%). If the estimator is not significant at 10% we say it is statistically insignificant

Up until now, we have restricted ourselves to testing whether  $\beta$  is equal to a constant (most often zero)

However, that is not always what we care about. Often we want to know the relative size of two  $\beta$ . E.g. does

$$\beta_2 = \beta_3$$

Or maybe we want to know if two effects cancel each other out. E.g. running (good for you) in a badly polluted area (bad for you). In this case maybe we want to know if

$$\beta_{running} + \beta_{pollution} = 0$$

We call these "linear combination" tests

How do we go about this process?

Theoretically, this is similar to process of testing the equality of two estimated means:

- **1** Define hypothesis:  $H_0: \beta_1 = \beta_2$   $H_1: \beta_1 \neq \beta_2$
- 2 Write the test statistic:

$$\frac{\hat{\beta}_1 - \hat{\beta}_2}{se(\hat{\beta}_1 - \hat{\beta}_2)}$$

3 Stop because we don't know how to calculate  $se(\hat{\beta}_1 - \hat{\beta}_2)$ We *can* calculate this value, but it is complex. There are two easier ways to do this in Stata.

First option:

1 Change variables in the regression so that the output tests equality directly

Lets say we have this model

$$lwage = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 exp + u$$

We want to test whether the returns to a year of junior college are the *same* as a year of four year college

1 Define our hypothesis

$$H_0: \beta_1 = \beta_2 \quad or \quad \beta_1 - \beta_2 = 0$$
$$H_1: \beta_1 - \beta_2 \neq 0$$

Now, we need to find an estimate of  $\beta_1 - \beta_2$  and the standard error of this object. We accomplish this through changing the variables in our regression:

Define  $\theta_1 = \beta_1 - \beta_2$   $lwage = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 exp + u$   $= \beta_0 + (\theta_1 + \beta_2) jc + \beta_2 univ + \beta_3 exper + u$   $= \beta_0 + \theta_1 jc + \beta_2 (jc + univ) + \beta_3 exper + u$  $= \beta_0 + \theta_1 jc + \beta_2 (tot coll) + \beta_3 exper + u$ 

So we can test if  $\theta$  is different from zero directly. **NOTE:** we could have also replaced *jc* with *totcoll* and we would also get the same test on  $\theta$ 

Option 2:

After regression output, write command

*test* Var1 == Var2

or

*test* 
$$2 * Var1 == Var2$$

The output will give you a test statistic and a p-value for the test you specify. This is useful to know for your problem sets, but doesn't help you understand what is going on. Suppose we want to test whether two parameters are *jointly* different from zero. Conceptually this is different from a linear combination test because we may want to know if two  $\beta$  are jointly significant even if they cancel each other out

E.g. if β<sub>2</sub> = -2 and β<sub>3</sub> = 2 both significant, a linear combination test of β<sub>2</sub> + β<sub>3</sub> would not reject the null, but an F-test of joint significance would reject the null

To do this we use an F-test. It's easiest to see how to do this from an example from a development data set on women's autonomy.

#### F-test

- Have data on women's autonomy, their current age, the age they got married, husband education, and urban or rural.
- Want to test whether age in general (*crage, mrage*) matters for autonomy

Steps:

1 Define hypotheses:

$$H_0: \beta_{crage} = 0$$
 &  $\beta_{mrage} = 0$ 

$$H_1: \beta_{crage} \neq 0 \text{ or } \beta_{mrage} \neq 0 \text{ or both}$$

Note the form of the null and alternative hypothesis!

2 Write down the two models the null hypothesis implies: Unrestricted model:

> autonomy =  $\beta_0 + \beta_1 mrage + \beta_2 crage + \beta_3 husbedu$ + $\beta_4 urban + u$

Restricted model:

 $autonomy = \beta_0 + \beta_3 husbedu + \beta_4 urban + u$ 

- We call the regression with the variables we are testing the **unrestricted model**.
- The regression without these variables is the restricted model
- Estimate both these models separately

- Unrestricted model: is the model were we allow  $\beta_{crage}$  and  $\beta_{mrage}$  to take on any value that best fits the data
- **Restricted model**: is the model where we force  $\beta_{crage}$  and  $\beta_{mrage}$  to be equal to zero
- For the F-test, we estimate both these models separately
  - We use the regression output from both models to calculate our test stat

3 Write our F-stat from the two regression outputs. Note: there are two equivalent ways to calculate the test statistic:

$$F = \frac{(SSR_R - SSR_{UR})/q}{SSR_{UR}/(n - k_{UR} - 1)}$$
$$F = \frac{(R_{UR}^2 - R_R^2)/q}{(1 - R_{UR}^2)/(n - k_{UR} - 1)}$$

Where

- q is the number of restrictions
- k is the number of variables in the unrestricted model
- F-stat is distributed  $F_{q,n-k-1}$  i.e. there are **two** degrees of freedom to track, the numerate (q) and the denomenator (n-k-1)

#### F-test

- 4 Compare the F-stat to the correct critical value found in the F-table. You will need to keep track of **both** numerator degrees of freedom (q) and denominator degrees of freedom (n - k<sub>UR</sub> - 1)
- 5 Interpret: We reject / don't reject the null hypothesis that both  $\beta_1$  and  $\beta_2$  are equal to zero

#### F-Test: Example

reg autonomy marr\_age curr\_age husb\_educ urban

Source	SS	df	MS
Model   Residual	949.690268 15809.3097	4 971	237.422567 16.2814724
Total	16759	975	17.1887179

Number of ob	os = 976
F( 4, 971	) = 14.58
Prob > F	= 0.0000
R-squared	= 0.0567
Adj R-square	ed = 0.0528
Root MSE	= 4.035

reg autonomy husb\_educ urban

Source	SS	df	MS	Number of obs = 976
	+			F(2, 975) = 14.54
Model	486.726699	2	243.363349	Prob > F = 0.0000
Residual	16317.306	975	16.7356985	R-squared = 0.0290
	+			Adj R-squared = $0.0270$
Total	16804.0327	977	17.1996241	Root MSE = 4.0909

Fill in the blanks:

$$F = \frac{\left(SSR_R - SSR_{UR}\right)/q}{SSR_{UR}/(n - k_{UR} - 1)}$$

Fill in the formula:

$$F = \frac{\left(SSR_R - SSR_{UR}\right)/q}{SSR_{UR}/(n - k_{UR} - 1)}$$

- $SSR_R = 16317.3$
- $SSR_{UR} = 15809.3$
- $R_R^2 = 0.029$
- $R_{UR}^2 = 0.057$
- *n* = 976
- $k_{UR} = 4$
- *q* = 2

$$F = \frac{(16317.306 - 15809.3097)/2}{15809.3097/(976 - 4 - 1)} = 15.6_{\{2,971\}}$$

F-Table

Appendix G Statistical Tables

Appendix G Statistical Tables

#### TABLE G.3b

set. Critical Values of the F Distribution

	Numerator Degrees of Freedom										
	- 1	.1	2	3	4	5	6	7	8	9	10
-	10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
	11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
D	12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.67
e n	13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.65	2.60
o	14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70		-
m	15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2,64	2.59	2.54
in	16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
a	17	4,45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.4
t	18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.4
0 T	19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.3
	20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.3
D e	20	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.3
g	21	4.30	3.44	3.05	2,82	2.66	2.55	2.46	2.40	2.34	2.3
r	23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.2
e	24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.2
ŝ		4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.2
	25	4.24	3.39	2.99	2.74	2.59	2.47	2.39	2.32	2.27	2.2
0 f	26 27	4.23	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.2
	28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.1
F	29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.1
e		4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.1
e d	30 40	4.17	3.23	2.84	2.61	2.45	2:34	2.25	2.18	2.12	2.0
0	60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.5
m	90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.5
	120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.5
			3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.
1	~	3.84	1 5.00	. 2.00	10.01	1		_		-	

~ 3%

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Example: The 5% critical value for numerator df = 4 and large denominator  $df (\infty)$  is 2.37. Source: This table was generated using the Stata<sup>®</sup> function invitrob.

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TABLE G.3a 10% Critical Values of the F Distribution

Numerator Degrees of Freedom											
200 1992 (1993		1	1. 1. 1. 1. 1. 1.	3		5	6	7	8	.9	10
D e n o m i n a t	10 11 12 13 14 15 16 17 18	3.29 3.23 3.18 3.14 3.10 3.07 3.05 3.03 3.01	2.92	2.73 2.66 2.61 2.56 2.52 2.49 2.46 2.44 2.42	2.61 2.54 2.48 2.43 2.39 2.36 2.33 2.31 2.29	2.52 2.45 2.39 2.35 2.31 2.27 2.24 2.22 2.20	2.46 2.39 2.28 2.24 2.21 2.18 2.15 2.13	2.41 2.34 2.28 2.23 2.19 2.16 2.13 2.10 2.08	2.38 2:30 2.24 2.20 2.15 2.12 2.09 2.06 2.04	2.35 2.27 2.21 2.16 2.12 2.09 2.06 2.03 2.00	2.32 2.25 2.19 2.14 2.10 2.06 2.03 2.00 1.98 1.96
o r	19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98 1.96	1.94
D e g r e	20 21 22 23	2.97 2.96 2.95 2.94	2.59 2.57 2.56 2.55 2.54	2.38 2.36 2.35 2.34 2.33	2.25 2.23 2.22 2.21 2.19	2.10 2.14 2.13 2.11 2.10	2.08 2.06 2.05 2.04	2.02 2.01 1.99	1.98 1.97 1.95 1.94	1.95 1.93 1.92 1.91	1.92 1.90 1.89 1.88
e s of F		2.90	2.53 2.52 2.51 2.50	2.33 2.32 2.31 2.30 2.29 2.28	2.18 2.17 2.17 2.16 2.15	2.09 2.08 2.07 2.06 2.06	2.01 2.00 2.00	1.96 1.95 1.94	1.91	1.89 1.88 1.87 1.87 1.87 1.86	1.87 1.86 1.85 1.84 1.83
r e d c n	30	2.88 2.84 2.79 2.79	2.49 2.44 2.39 5. 2.36	2.28 2.23 2.18 2.15 2.13	2.14 2.09 2.04 2.01	2.05 2.00 1:95 1.9	5 1.98 0 1.93 5 1.8 1 1.8	8 1.93 3 1.87 7 1.82 4 1.78 2 1.7	1.83 1.77 1.77 1.74 7 1.72	1.85 1.79 1.74 1.70 1.68	-
	00	2.7		2.08	1.94	1.8	-	_	2 1.67	-	1.60

Example: The 10% critical value for numerator df = 2 and denominator df = 40 is 2.44. Source: This table was generated using the Stata<sup>®</sup> function invfprob.

## Hypothesis Testing & CI Overview

#### Standard Deviation vs. Standard Error:

Test Type	Variance of X or u	StDev of X or u	Var. of Estimator	StDev Estimator
$\mu$ , known $\sigma^2$	$\sigma_X^2 = E[(X_i - E[X_i])^2]$	$\sqrt{\sigma_X^2}$	$\frac{\sigma_X^2}{n}$	$\sqrt{\frac{\sigma_X^2}{n}}$
Unknown $\sigma^2$	$s_X^2 = \sum_i (X_i - \bar{X})^2$	$\sqrt{s_X^2}$	$\frac{s_X^2}{n}$	$\sqrt{\frac{s_X^2}{n}}$
$Proportion\ (p)$	Var(X) = p(1-p)	$\sqrt{p(1-p)}$	$\frac{p(1-p)}{n}$	$\sqrt{\frac{p(1-p)}{n}}$
$\beta$ , known $\sigma^2$	$\sigma^2$	σ	$\frac{\sigma^2}{SST_{x_j}(1-R_j^2)}$	$\sqrt{\frac{\sigma^2}{SST_{x_j}(1-R_j^2)}}$
Unknown $\sigma^2$	$\left(\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-k-1}\right)$	$\left(\sqrt{\hat{\sigma}^2}\right)$	$\frac{\partial^2}{SST_{x_j}(1-R_j^2)}$	$\sqrt{\frac{\hat{\sigma}^2}{SST_{x_j}(1-R_j^2)}}$

## T-Stats and Distributions Overview

#### Fill in the blanks!

Test Type	Test Statistic	Distribution
Population mean		
e.g. $H_0: \mu = \mu_0$		
Difference in population means		
e.g. $H_0: \mu_1 - \mu_2 = \mu_0$		
Population proportion		
e.g. $H_0: p = p_0$		
Difference in population proportions		
e.g. $H_0: p_1 - p_2 = p_0$	$z = \frac{(\hat{p}_1 - \hat{p}_2) - p_0}{\sqrt{\frac{\hat{p}(1 - \hat{p})}{n_1} + \frac{\hat{p}(1 - \hat{p})}{n_2}}}$	$z \sim N(0, 1)$
True regression parameter (k other vars)		
e.g. $H_0: \beta = \beta_0$		
Multiple restrictions in regression		
(q  restrictions,  k  total variables in UR model)	$F = \frac{\left(R_{UR}^2 - R_R^2\right)/q}{\left(1 - R_{UR}^2\right)/(n-k-1)}$	$F \sim F_{q,n-k-1}$

## T-Stats and Distributions Overview

Test Type	Test Statistic	Distribution
Population mean		
e.g. $H_0: \mu = \mu_0$	$t = rac{ar{x} - \mu_0}{\sqrt{rac{s^2}{n}}}$	$t \sim t_{n-1}$
Difference in population means		
e.g. $H_0: \mu_1 - \mu_2 = \mu_0$	$t = \frac{(\bar{x}_1 - \bar{x}_2) - \mu_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	$t \sim t_{n_1+n_2-2}$
Population proportion		
e.g. $H_0: p = p_0$	$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$	$z \sim N(0, 1)$
Difference in population proportions		
e.g. $H_0: p_1 - p_2 = p_0$	$z = \frac{(\hat{p}_1 - \hat{p}_2) - p_0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}}$	$z \sim N(0, 1)$
True regression parameter ( $k$ other vars)		
e.g. $H_0: \beta = \beta_0$	$t = rac{\hat{eta} - eta_0}{SE(\hat{eta})}$	$t \sim t_{n-k-1}$
Multiple restrictions in regression		
(q  restrictions, k  total variables in UR model)	$F = \frac{(R_{UR}^2 - R_R^2)/q}{(1 - R_{UR}^2)/(n - k - 1)}$	$F \sim F_{q,n-k-1}$

## Practice

Fill in the blanks:

#### . reg lwage educ exper female nonwhite

Source	SS	df	MS		Number of obs	
Model   Residual   	182.711923 489.864945 672.576867	-	45.6779807 .245546338 .336456662		F( 4, 1995) Prob > F R-squared Adj R-squared Root MSE	= 0.0000
lwage	Coef.	Std. E		P> t	[95% Conf.	
educ exper female nonwhite cons	.1166997 .0108872 2533177 0374311 1.061903	.00531 .00086 .02221	591 12.53 198 -11.40 1.20	0.000	.1062756 .0091827 0985117 .9130514	.1271237 .0125917 .0236495 1.210756