

EEP/IAS 118 - Introductory Applied
Econometrics, Lecture 8

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This Lecture

Topics

- Scaling & Standardized Effects
- Confidence Intervals for Predictions
- Choice between non-nested models

Scaling Variables

Often times the units that variables come in are not the most useful for interpretation or analysis.

- Rescaling monetary units - \$ thousands, \$ billions, etc.
- Distance per second into distance per hour

Example:

$$\widehat{sleep} = 3315.574 - 12.189educ + 2.7454age$$

Where sleep is measured in minutes per night. Here, $\hat{\beta}_{educ}$ is interpreted:

- One more year of education is estimated to decrease predicted sleep by 12.189 minutes per week, holding age constant

Scaling Variables

$$\widehat{sleep} = 3315.574 - 12.189educ + 2.7454age$$

Lets say we instead want to change the dependent variable to be measured in hours rather than minutes.

- Do this simply by changing our y variable into $\tilde{y} = \frac{y}{60}$

How would this change our $\hat{\beta}$?

- The new β_{educ} estimate would be $\frac{12.189}{60} = 0.2$ hours per night

The entire regression result changes to this:

$$\widehat{sleep} = 55.260 - .2032educ + .0458age$$

Scaling Variables

In general, when we re-scale the outcome variable by α

$$\begin{aligned}\tilde{y} &= \tilde{\beta}_0 + \tilde{\beta}_1 x_1 + \dots + \tilde{\beta}_k x_k + u \\ \alpha y &= \alpha \beta_0 + \alpha \beta_1 x_1 + \dots + \alpha \beta_k x_k + u\end{aligned}$$

In the above example, $\alpha = \frac{1}{60}$, so the new $\hat{\beta}$ s will be divided by 60 too.

- **Note:** nothing else about the regression will change (R^2 , t-stats, p-values, etc.)

Scaling Variables

Let's say instead we rescale an independent x variable:

- Rescale education to be in units of half-years (6 months) - i.e. we multiply $educ$ by 2
- The new regression would give us:

$$\widehat{sleep} = 3315.574 - 6.095educ + 2.7454age$$

- Only the coefficient on the independent variable we modified has changed

Scaling Variables

In general, if we scale x by α , the equation becomes:

$$\begin{aligned}y &= \beta_0 + \tilde{\beta}_1 \tilde{x}_1 + \dots + \beta_k x_k + u \\ &= \beta_0 + \frac{\beta_1}{\alpha} (\alpha x_1) + \dots + \beta_k x_k + u\end{aligned}$$

- In the above example, we had $\alpha = 2$, which meant we had to scale our estimate of $\hat{\beta}_{educ}$ by $\frac{1}{2}$.

Standardizing Variables

Up until now we've been considering cases where we want to change the units of a variable into units that are more useful

- But what if we don't want units at all? Why would we want this?
 - Want to compare the relative effects of two variables that don't have the same unit - e.g. education and SAT score on income
- This is useful for many economic models: Hedonic Price Model

Hedonic Price Model

Idea behind Hedonic Price Model:

- We want to measure “Willingness to Pay” (WTP) for certain amenities:
 - Environmental amenities (clean water, clean air, parks, ect.)
 - House Characteristics (school district, local pollution, etc.)
- These can be difficult to measure, as most people are never asked to explicitly “buy” these goods
- How do we measure their value:
 - ① Directly ask: “What is your WTP?” via survey
Problem: Question framing important, people will inflate / deflate values because choice is not real
 - ② Revealed Preference: you reveal your preference for amenities via the value you paid to obtain them

Hedonic Price Model

Two common ways to do this:

- 1 **Travel cost method:** Used for value of fishing / beaches - you can infer the value of these amenities by how much people pay to travel to access them (especially over closer locations without these amenities)
- 2 **Hedonic Price:** When you choose a place to live, your WTP for the house reveals your preference for the value of all the amenities the house has access to

Hedonic Price:

$$price = f(\#rooms, size\ yard, \dots, pollution, crime, school\ quality)$$

Hedonic Price Model

$$price = f(\#rooms, size\ yard, \dots, pollution, crime, school\ quality)$$

Would use data from housing sales, with price, and all information about the house and location we can and then we would run a linear model:

$$price = \beta_0 + \beta_1 NO_2 + \beta_2 crime + \beta_3 rooms \dots$$

Hedonic Price Model

```
. reg price nox crime dist rooms lowstat stratio, beta
```

Source	SS	df	MS
Model	3.0150e+10	6	5.0250e+09
Residual	1.2675e+10	499	25401468.7
Total	4.2826e+10	505	84803032

price	Coef.	Std. Err.	t	P> t
nox	-1757.656	331.4642	-5.30	0.000
crime	-80.57672	30.47786	-2.64	0.008
dist	-1202.372	170.5011	-7.05	0.000
rooms	4412.584	415.8469	10.61	0.000
lowstat	-519.7665	48.41627	-10.74	0.000
stratio	-998.834	115.819	-8.62	0.000
_cons	34431.7	4732.075	7.28	0.000

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But we want to *compare* these coefficients!

- What do consumers value more - crime or pollution?
- Problem is that crime and pollution have vastly different ranges

We can do this by *standardizing* the variables in the regression

Standardize Variables

Standardizing means we will compare how a one standard deviation increase in x_1 affects y to how a one Stdev increase in x_2 affects y

We do this by transforming all our variables by subtracting their mean and dividing by the standard deviation:

$$\tilde{y} = \left(\frac{y - \bar{y}}{\hat{\sigma}_y} \right)$$
$$\tilde{x} = \left(\frac{x_1 - \bar{x}_1}{\hat{\sigma}_{x_1}} \right)$$

This should look familiar - this is what we do with our t-stats! Idea is that we put all the variables on the same scale. Then we can compare relative effects.

Standardize Variables

Once we standardize all the units, re-running the regression produces:

$$\left(\frac{y - \bar{y}}{\hat{\sigma}_y} \right) = \frac{\hat{\sigma}_{x_1}}{\hat{\sigma}_y} \hat{\beta}_1 \left(\frac{x_1 - \bar{x}}{\hat{\sigma}_{x_1}} \right) + \frac{\hat{\sigma}_{x_2}}{\hat{\sigma}_y} \hat{\beta}_2 \left(\frac{x_2 - \bar{x}_2}{\hat{\sigma}_{x_2}} \right)$$

- The new parameters will be equal to the old parameters scaled by $\frac{\hat{\sigma}_{x_1}}{\hat{\sigma}_y}$
- This is called the “standardized coefficient” or the “beta coefficient”
- Note there is no β_0 because it will be zero (why?)
- In Stata we can produce these coefficients with the “beta” option (because transforming each variable is a pain)

Standardize Variables

```
. reg price nox crime dist rooms lowstat stratio, beta
```

Source	SS	df	MS	Number of obs =	506
Model	3.0150e+10	6	5.0250e+09	F(6, 499) =	197.82
Residual	1.2675e+10	499	25401468.7	Prob > F =	0.0000
				R-squared =	0.7040
				Adj R-squared =	0.7005
				Root MSE =	5040
Total	4.2826e+10	505	84803032		

price	Coef.	Std. Err.	t	P> t	Beta
nox	-1757.656	331.4642	-5.30	0.000	-.2210981
crime	-80.57672	30.47786	-2.64	0.008	-.0751639
dist	-1202.372	170.5011	-7.05	0.000	-.2749917
rooms	4412.584	415.8469	10.61	0.000	.3366601
lowstat	-519.7665	48.41627	-10.74	0.000	-.4085311
stratio	-998.834	115.819	-8.62	0.000	-.2349146
_cons	34431.7	4732.075	7.28	0.000	.

- One SD increase in pollution leads to 0.22 SD decrease in prices
- One SD increase in crime leads to a 0.07 SD decrease in prices

Confidence Intervals for y

There are some instances where we may care about the predicted value of the dependent variable y

We know that the estimated regression give us \hat{y} which is our best guess for y for and given x . However, \hat{y} is a random variable (just like $\hat{\beta}$) and therefore has uncertainty.

- We can quantify this uncertainty and create a confidence interval for \hat{y} for any specific combination of x_j

Confidence Intervals for y

However, there are two types of CI that we may want to calculate:

- 1 A confidence interval for the **average** y given x_1, \dots, x_k
- 2 A confidence interval for a **particular** y given x_1, \dots, x_k

You can think of the difference as being the answer to these two questions:

- 1 How uncertain are we about the average income for this type of person?
- 2 If we asked a person of this type their income, what range would cover 95% of responses

Confidence Intervals for average y

Recall that regression gives us an estimate of y given x :

$$\hat{\mathbb{E}}[y|x_1, x_2, x_3] = \hat{y} = \hat{\beta}_0 + \hat{\beta}_1x_1 + \hat{\beta}_2x_2 + \hat{\beta}_3x_3$$

- If we want the best estimate for a particular value of x_j , we just plug those values into the equation
- To get a CI, then we only need to find the stand error for this prediction
- Recall that β_0 takes on the predicted value of y when all the x_j are zero

$$\hat{\beta}_0 = \hat{E}(y|x_1 = 0, x_2 = 0, x_3 = 0)$$

Confidence Intervals for average y

$$\hat{\beta}_0 = \hat{E}(y|x_1 = 0, x_2 = 0, x_3 = 0)$$

- Therefore, if we transform our x_j by subtracting the values (α_j) for which we want a prediction:

$$y = \beta_0 + \beta_1(x_1 - \alpha_1) + \beta_2(x_2 - \alpha_2) + \beta_3(x_3 - \alpha_3)$$

Then

$$\hat{\beta}_0 = \hat{E}(y|x_1 = \alpha_1, x_2 = \alpha_2, x_3 = \alpha_3)$$

When we run the regression with these transformed variables, $\hat{\beta}_0$ will then be best prediction and Stata will produce the correct SE

Confidence Intervals for average y

Process Summary for CI on average y :

- 1 Generate new variables: $\tilde{x}_j = x_j - \alpha_j$.
- 2 Run the regression of: $y = \tilde{\beta}_0 + \tilde{\beta}_1 \tilde{x}_1 + \dots + \tilde{\beta}_k \tilde{x}_k + \tilde{u}$
- 3 Then $\hat{\mathbb{E}}[y|x_1 = \alpha_1, \dots, x_k = \alpha_k] = \tilde{\beta}_0$ and the standard error for this estimate is $SE(\tilde{\beta}_0)$.
- 4 Plug these values into the formula for confidence intervals and interpret.

$$[\tilde{\beta}_0 - c \cdot SE(\tilde{\beta}_0) , \tilde{\beta}_0 + c \cdot SE(\tilde{\beta}_0)]$$

Confidence Intervals for average y : Example

For an example, let's use Woolridge's birthweight data. Let's say we want to find a prediction for average birthweight for babies with family income of \$14,500 ($\ln(14.5) = 2.674$), mothers with 12 years of education, and with 2 older siblings ($parity = 3$)

Running the standard regression:

$$\begin{aligned}\widehat{bwght} &= 105.66 + 2.13\ln(faminc) + 0.317meduc + 1.53parity \\ \hat{y} &= 105.66 + 2.13(2.674) + .317(12) + 1.53(3) \\ &= 119.75 \text{ ounces}\end{aligned}$$

Which is our best guess for $\hat{y}_{faminc=14.5,meduc=12,parity=3}$

Confidence Intervals for average y : Example

To get the SE of this prediction, we run:

$$bwght = \beta_0 + \beta_1(lfaminc - 2.674) + \\ \beta_2(meduc - 12) + \beta_3(parity = 3) + u$$

bwght	Coef.	Std. Err.	t	P> t
lfaminc_0	2.131266	.6505986	3.28	0.001
meduc_0	.3171976	.2519682	1.26	0.208
parity_0	1.526144	.6119145	2.49	0.013
_cons	119.6405	1.006928	118.82	0.000

Note, how now the “cons” takes on the predicted value and has a standard error!

Confidence Intervals for average y : Example

Using this output, the 95% confidence interval for the average birthweight for babies given family income of \$14,500 ($\ln(14.5) = 2.674$), mothers with 12 years of education, and with 2 older siblings ($parity = 3$) is:

$$[119.64 - 1.96(1.007), 119.64 + 1.96(1.007)] = [117.6653, 121.6158]$$

Confidence Intervals for a particular y

Now let's turn to how we can create a confidence interval of y for a *particular* individual with certain x .

- Again, this is different (larger) than our CI for the *average* y in a sub-population
- This is because we need to account for both the variance in our calculation of \hat{y} as well as the variance unobserved error term u

Confidence Intervals for a particular y

Let's see how to think about this using our example. Let $bwght^0$ denote the value for which we want to construct a confidence interval:

$$bwght^0 = \beta_0 + \beta_1 lfaminc^0 + \beta_2 meduc^0 + \beta_3 parity^0 + u^0$$

Our best prediction of $bwght^0$ is \widehat{bwght}^0 , where

$$\widehat{bwght}^0 = \hat{\beta}_0 + \hat{\beta}_1 lfaminc^0 + \hat{\beta}_2 meduc^0 + \hat{\beta}_3 parity^0$$

Now there is some error associated with using \widehat{bwght}^0 to predict $bwght^0$:

$$\begin{aligned} \hat{u}^0 &= bwght^0 - \widehat{bwght}^0 = \beta_0 + \beta_1 lfaminc^0 + \beta_2 meduc^0 + \beta_3 parity^0 + u^0 \\ &\quad - \hat{\beta}_0 + \hat{\beta}_1 lfaminc + \hat{\beta}_2 meduc + \hat{\beta}_3 parity \end{aligned}$$

Confidence Intervals for a particular y

To get a confidence interval, we need to quantify the variance of the error in this prediction:

$$\begin{aligned} \text{Var}(\hat{u}^0) &= \text{Var}(bwght^0 - \widehat{bwght}^0) \\ &= \text{Var}(\beta_0 + \beta_1 lfaminc^0 + \beta_2 educ^0 + \beta_3 parity^0 + u^0 - \widehat{bwght}^0) \\ &= \text{Var}(\widehat{bwght}^0) + \text{Var}(u^0) \\ &= \text{Var}(\widehat{bwght}^0) + \sigma^2 \\ \widehat{\text{Var}}(\hat{u}^0) &= \text{Var}(\widehat{bwght}^0) + \hat{\sigma}^2 \\ &= \text{Var}(\widehat{bwght}^0) + \frac{\sum \hat{u}_i^2}{n - k - 1} = \text{Var}(\widehat{bwght}^0) + \frac{SSR}{n - k - 1} \end{aligned}$$

Confidence Intervals for a particular y

$$\text{Var}(\widehat{bwght}^0) + \frac{SSR}{n - k - 1}$$

There are two sources of variation in \hat{u}^0

- 1 The sampling error in \widehat{bwght}^0 which arises because we have estimated the population parameters (β).
 - 2 The variance of the error in the population (u^0).
- Compute the $\text{Var}(\widehat{bwght}^0)$ exactly as before
 - Second we can compute $\frac{SSR}{n-k-1}$ from our regression output
 - Then the 95% confidence interval for $bwght^0$:

$$\hat{y} \pm 1.96 \cdot se(\hat{u}^0)$$

Confidence Intervals for a particular y : Summary

- 1 Generate new variables: $\tilde{x}_j = x_j - \alpha_j$.
- 2 Run the regression of: $y = \tilde{\beta}_0 + \tilde{\beta}_1\tilde{x}_1 + \dots + \tilde{\beta}_k\tilde{x}_k + \tilde{u}$
- 3 Then $\hat{\mathbb{E}}[y|x_1 = \alpha_1, \dots, x_k = \alpha_k] = \tilde{\beta}_0$ and the standard error for this estimate is $SE(\tilde{\beta}_0)$.
- 4 Get an estimate for the variance of $\hat{u} = \hat{\sigma}^2$ from the Stata output.
- 5 Compute the standard error: $\sqrt{SE(\tilde{\beta}_0)^2 + \hat{\sigma}^2}$.
- 6 Plug these values into the formula for confidence intervals and interpret.

Choice Between Non-Nested Models

You've been asked to do the following in past problems:

- 1 Deciding if one of your x variables is significant \Rightarrow t-test
- 2 Deciding if multiple variables *together* are significant \Rightarrow F-test.

These tests compare *nested* models

- Nested models are cases where one equation is just a special case of the other (e.g. fixing β_3 and $\beta_4 = 0$)

How do we compare *non-nested* models?

- Use **Adjusted R^2**

Adjusted R^2 : Comparing Non-nested Models

Regular R^2 is a measure of “goodness of fit”, so why not just use that?

- R^2 will always (weakly) increase when you add more variables to the regression
- Not useful to choosing which model is better, more complex one will always win

Therefore, we use Adjusted R^2 which adds a penalty for each additional variable added to the model

Adjusted R^2

The formula for adjusted R^2 is:

$$1 - \frac{SSR/(n - k - 1)}{SST/(n - 1)}$$

Adding variables now has two effects:

- 1 The SSR in the numerator will always (weakly) decrease with an additional variable
- 2 However, k will also increase (making the numerator larger)

Therefore, the effect on the adjusted R^2 from adding an additional variable to the regression will depend on if the extra explanatory power is larger than the penalty

Adj R^2 : Two Models of Sleep

When does Adj R^2 come in handy:

- Choosing a functional form for the right hand side variables can be difficult
- A common example is a choice between $\log(x)$ and a quadratic x and x^2
- Both can be reasonable choices and it is difficult to eyeball which is better

Adj R^2 : Two Models of Sleep

reg sleep lnage

Source	SS	df	MS			
Model	891303.042	1	891303.042	Number of obs =	706	
Residual	138348533	704	196517.802	F(1, 704)	= 4.54	
				Prob>F	= 0.0335	
				R-squared	= 0.0064	
				Adj R-squared	= 0.0050	
				Root MSE	= 443.3	
Total	139239836	705	197503.313			
sleep	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
lnage	122.9174	57.71672	2.13	0.034	9.599897	236.2349
_cons	2821.777	209.4207	13.47	0.000	2410.613	3232.941

Source	SS	df	MS			
Model	2039007.98	2	1019503.99	Number of obs =	706	
Residual	137200828	703	195164.762	F(2, 703)	= 5.22	
				Prob>F	= 0.0056	
				R-squared	= 0.0146	
				Adj R-squared	= 0.0118	
				Root MSE	= 441.77	
Total	139239836	705	197503.313			
sleep	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
age	-21.4904	11.73674	-1.83	0.068	-44.53366	1.552851
agesquared	.3011932	.140117	2.15	0.032	.0260954	.576291
_cons	3608.03	230.6457	15.64	0.000	3155.193	4060.867

Which do we prefer? Look at $Adj - R^2$

Choice Between y and $\ln(y)$

Rather than trying to choose between what x to include in a model, what if we are trying to choose between different functional forms of y ? A common example is the choice between y and $\ln(y)$

A natural choice might be to run both regressions:

$$y = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k + \hat{u}$$

$$\ln(y) = \hat{\alpha}_0 + \hat{\alpha}_1 x_1 + \dots + \hat{\alpha}_k x_k + \hat{u}$$

And then look at the R^2 of each model to decide the best fit. But this is **wrong**

Choice Between y and $\ln(y)$

How can we see the problem? Remember that $R^2 = \text{corr}(y, \hat{y})^2$ so what we're actually comparing is:

$$\text{corr}(y, \hat{y})^2 \quad \text{to} \quad \text{corr}(\ln(y), \widehat{\ln(y)})^2$$

- Comparing the R^2 for each model isn't an apples to apples comparison
- We need to do something else

Choice Between y and $\ln(y)$

Process to choose:

- 1 Estimate the log model: $\ln(y) = \hat{\alpha}_0 + \hat{\alpha}_1 x_1 + \dots + \hat{\alpha}_k x_k + u$
- 2 Predict: y from the log model: $\hat{y} = e^{\widehat{\ln(y)}} e^{\frac{\hat{\sigma}^2}{2}}$
- 3 Find the correlation and square it (to get alternative R_{log}^2): y and the \hat{y} from the log model. This gives us an alternative R^2 .
- 4 Estimate the linear model (to get R_{lin}^2):
 $y = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k + u$ and get its R^2
- 5 Compare R_{lin}^2 to R_{log}^2 and choose the higher one

Note: that to predict \hat{y} from $\widehat{\ln(y)}$, you need to raise to the exponential *and* multiply by $e^{\frac{\hat{\sigma}^2}{2}}$ (where $\hat{\sigma}^2$ is found in the regression output under MS residual)

Choice Between y and $\ln(y)$: Example

```
. reg lprice llotsize lsqrft bdrms
```

Source	SS	df	MS
Model	5.15504028	3	1.71834676
Residual	2.86256324	84	.034078134
Total	8.01760352	87	.092156362

```
Number of obs =      88  
F( 3, 84) = 50.42  
Prob > F      = 0.0000  
R-squared     = 0.6430  
Adj R-squared = 0.6302  
Root MSE     = .1846
```

lprice	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
llotsize	.1679667	.0382812	4.39	0.000	.0918404	.244093
lsqrft	.7002324	.0928652	7.54	0.000	.5155597	.8849051
bdrms	.0369584	.0275313	1.34	0.183	-.0177906	.0917074
_cons	-1.297042	.6512836	-1.99	0.050	-2.592191	-.001893

Choice Between y and $\ln(y)$: Example

```
. reg price lotsize sqrft bdrms
```

Source	SS	df	MS
Model	617130.701	3	205710.234
Residual	300723.805	84	3580.0453
Total	917854.506	87	10550.0518

```
Number of obs =      88  
F( 3, 84) = 57.46  
Prob > F      = 0.0000  
R-squared     = 0.6724  
Adj R-squared = 0.6607  
Root MSE     = 59.833
```

price	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
lotsize	.0020677	.0006421	3.22	0.002	.0007908 .0033446
sqrft	.1227782	.0132374	9.28	0.000	.0964541 .1491022
bdrms	13.85252	9.010145	1.54	0.128	-4.065141 31.77018
_cons	-21.77031	29.47504	-0.74	0.462	-80.38466 36.84405

Choice Between y and $\ln(y)$: Example

Process:

- 1 Estimate the log model:
reg log(price) log(lotsize) log(sqrft) bdrms
- 2 Get your predictions from this regression: predict lpricehat
- 3 Predict: y from the log model: gen
pricehat= $\exp(\text{lpricehat}) * \exp(.034078134/2)$
- 4 Find the correlation and square it: correl price pricehat
(= 0.7377)
- 5 Find the R^2 from the linear regression ($R^2 = 0.6724$)
- 6 Compare: For predicting price, the log model is notably better.

Review

- 1 One parameter: use t-stat and test significance
- 2 Multiple parameters: use F-test
- 3 Choosing between two non-nested models: Adj R^2
- 4 Choosing between different functional forms for y (y and $\ln(y)$), use process above